In this high-tech and globally competitive society, it is becoming more and more important that all citizens be confident in their ability to do mathematics. Knowledge of mathematics is an important skill necessary to succeed in today’s world. All students deserve equal access to learning math, and teachers must make the effort to ensure this. The National Council of Teachers of Mathematics (NCTM, 2000), in their revised and updated standards, identified “equity” as their first principle for school mathematics: “Excellence in mathematics education requires—high expectations and strong support for all students” (p. 11). NCTM has noted, “Equity requires accommodating differences to help everyone learn mathematics” (p. 13). The NCTM has taken a prominent stand that as educators we must take an equity-for-all-students approach to teaching mathematics. All students have the right to learn math and feel confident in their ability to do math. Teachers must see to it that “mathematics can and will be learned by all students” (p. 13).

In this article we have listed 20 best practices or strategies for teaching mathematics to reach all students. These teaching strategies may be effective with English Language Learners (ELL), special education students, or with those served in mainstream classes. With diverse learners, the use of the multimodal approach that incorporates multiple intelligences caters to students’ short attention spans, as they are not expected to merely sit still to learn the material. All students can learn math through acting out math problems; for instance, go on Internet fieldtrips with a typically able peer and manipulate tangible objects that help them to concretize abstract concepts. English Language Learners often need specially designed instruction in English. By using the strategies and approaches in this article, teachers can help support the teaching of language acquisition while teaching the content area. In reality, these strategies really are just best practice for the teaching of mathematics in general.

**Teach vocabulary using realia and demonstration.** Teachers can use real objects such as coupons, fruit, patterned blocks, beans, Popsicle® sticks, marbles, buttons, or M&Ms® as manipulatives in demonstrating math concepts. This can reinforce the number of sentences visually, and diverse-needs learners can also practice English and
Relate math problems and vocabulary to prior knowledge and background.

Especially for teaching students from diverse backgrounds, teachers can assess the current strategies used by their students to learn math. For instance, Chinese students may be familiar with the use of an abacus to do their calculations. Teachers can perhaps ask these students to show the others in class how an abacus is used. Honoring and recognizing students’ knowledge will boost their self-esteem because students will feel that they, too, have something to contribute to the learning process despite their limited English abilities.

Students with learning disabilities in the area of math can be powerful teachers to other students when they explain how they learned to accomplish specific skills or concepts (Mastropieri et al., 2001, p. 23). In addition, teachers can prompt students to talk about their experience in learning some of the math concepts in their country. By capitalizing on students’ prior knowledge, teachers who are empathetic to their students’ needs and backgrounds bridge the new knowledge to the old, making learning new math concepts more manageable for these students. In some cases, by understanding how students solve problems, teachers can troubleshoot or fine tune the individual student’s process and make him or her more efficient learners.

Apply problems to daily life situations.

Creative teachers can use a variety of ways to coach students when applying problems to daily life situations. For example, teachers can use restaurant take-out menus to teach students multiplication and division. Not only do such activities link students with real-life situations but they also create a fun learning environment. Such an environment will also promote English language acquisition for the non-native English students and model problem-solving techniques aloud for special education students. Krashen’s (1985) metaphorical use of “affective filter” in his affective filter hypothesis reinforces the idea that teachers can lower the affective filter by fostering a spirit of mutual respect, high expectations, and cooperative learning. By using real-world practice activities, the goal of generalizing skills learned in class to their lives becomes more attainable.

Use manipulatives to make problems concrete. Best practices in special education call for you to teach concepts with concrete examples, and once the vocabulary and process is understood, then move to more abstract problems (Mercer & Mercer, 1998). The same practice is equally effective when considering the needs of Limited English Proficiency (LEP) or learners from diverse backgrounds. Teachers can obtain commercial manipulatives, or their students can make their own manipulatives (e.g., paper money, buttons, blocks, rods, tangrams, pattern blocks, algebra tiles). The use of manipulatives provides teachers with a great potential to use their creativity to do further work on the math concepts instead of merely relying on worksheets. Consequently, students are learning math in an enjoyable way, making connections between the concrete and the abstract.

Encourage drawings to translate and visualize word problems. The Natural Approach (Krashen, 1985; Terrell, 1981) is used extensively with ELL students. One of the four principles of this approach is that the teacher understands that the student will need to have a silent period before being expected to speak English. One of the subsequent strategies of the Natural Approach is to allow students, especially those at the beginning level of their English language developmental stage, to use drawings and symbols in solving some of the math problems. The same techniques are employed with students with learning disabilities to allow them to process auditory information before making a verbal response. In fact, as a comprehension strategy, teachers can use students’ drawings and verbal rehearsals as testimony of their understanding of math concepts. This approach can alleviate frustrations for both teachers and students.
6 Have ELL/special education students pair with typical students for computer/cooperative activities.

According to Krashen, language is acquired in an “amazingly simple way—when we understand messages” (1985, p. vii). He termed this understandable language as “comprehensible input.” Peer interaction between native and non-native speakers of English is one means of promoting “comprehensible input” or understandable language. In peer interaction, students are observed to have used four communication strategies that contribute to the occurrence of comprehensible input:

1. embedding language within a meaningful context,
2. modifying language presented to non-native peers,
3. judiciously using paraphrase and repetition, and
4. consistently negotiating meaning (Diaz-Rico & Weed, 1995).

When teachers make diverse pairings, students accomplish content- or language-task goals as well as math goals through the use of computers. Collaborative work between the pair will promote student acquisition of the English language and facilitate inclusion through the learning of content areas such as math, science, social studies (Howell, 1996, p. 60). Students may be encouraged to take group Internet field trips or work on a piece of software in a cooperative fashion.

7 Encourage children to think aloud when solving word problems, and have students give oral explanations of their thinking, leading to solutions. The think aloud technique was a research tool originally used by psychologists. This research instrument is now popular with researchers in the language and reading fields. When a person thinks aloud they are verbalizing the concepts, processes, or metacognitive checks that enable them to solve problems. According to Chamot & O’Malley (1989), metacognitive knowledge includes awareness of task demands, of one’s own approach to learning and experiences with similar tasks, and of appropriate strategies for the task. Encouraging children to think aloud when solving problems helps teachers pinpoint students’ difficulties in solving math problems. In addition, it can also help teachers instill in their students the metacognitive knowledge and strategies when learning math concepts. Most times, in verbalizing step-by-step how a math problem is solved, students can self-correct their mistakes. Similarly, this process allows peer corrections to occur.

8 Have students write original word problems to exchange with classmates.

When students write their own original word problems, the teacher can use them as part of review games among cooperative groups. For example, teachers can divide students into groups of three or four and have them collaboratively write word problems. After the group has generated several math problems, the group members take turns writing and adding to the word problems. The group that comes up with the most difficult problem but displays clear and well-written word problems wins. This activity can reinforce students’ writing and reading skills.

9 Explain directions clearly, and repeat key terms. Diaz-Rico and Weed (1995) asserted that “the difficulties that language minority students have with the language of mathematics lie in four major areas: vocabulary skills, syntax, semantics, and discourse features” (p. 137). These same language difficulties are evident in many students with learning disability profiles. Teachers who are sensitive to student language difficulties will explain directions clearly and repeat key terms. There are several ways that teachers can do this. Common classroom directions such as “hand work in” or “work quietly” should be written in bold letters and/or illustrated by drawings, cue cards, and diagrams posted on the classroom wall. Teachers should repeat routine-related directions every day or have students take turns in repeating directions so that all students will understand what is expected of them. As far as the learning of math formulae and symbols, teachers can assign group projects that elicit students’ help in illustrating math concepts.
Encourage students to follow the four-step problem-solving process.

Students should be encouraged to use Polya’s (2004) four-step problem-solving process and to write their thought processes as they go about solving problems. This technique is a crucial part of the NCTM’s (1989, 2000) Standards. Polya’s method directs students to solve math problems by doing the following:

- Read and understand the problem. They may write the problem in simpler terms.
- Develop a student-generated strategy for solving the problem, and discuss how they arrived at this strategy.
- Carry out their strategy/plan, and show all work justifying their answer.
- Look back and check to see that their solution appears to be reasonable.

Also, teach problem-solving strategies to students: working backward, drawing a picture, making a simpler problem, looking for a pattern, learning by trial and error, acting out, using a table. These strategies can enrich and empower students mathematically as they problem solve. Like all strategies, this one requires teacher direction and modeling at first, then verbal rehearsal and practice on simpler problems before moving to more difficult ones or generalization (Mercer & Mercer, 1998).

Realize that not all math notations are necessarily universal. Although math is considered by many as an international language, students from South America and many European countries write a period in large numbers where comma is written, and where their decimal mark is a comma, our decimal mark is a period. That is, our number 4.547 is interpreted by these students as 4,547 or vice versa. Consequently, students with visual perceptual difficulties may not accurately distinguish between the two notations regardless of their background or ethnicity. Another example of differing notational systems is the methods used to divide numerals. Students from South American and the Caribbean may display their work for division problems quite differently from the U.S. manner.

Example: 25,000 divided by 5
U.S.: 5) 25000
Haiti: 25,000 (5

Most of the world uses the metric system; therefore math concepts based on feet, inches, miles, pounds, ounces, cups, pints, quarts, and so forth have to be relearned and practiced. Math concepts that are based on money and time are not universal. American coins may confuse newcomers with a relatively large nickel outweighing a coin of twice the value and by not having the value written on the coins in numbers.

Group students heterogeneously during cooperative learning. Cooperative learning has positive results in the education of students from diverse backgrounds (Kagan, 1989). According to Cazden (1988), classrooms that emphasize individual performance, teacher-centered learning, and little or no student control of participation may be “culturally incongruent” with the backgrounds of many students. Not only does cooperative learning restore a sense of comfort in a school setting where there are students with differing needs, it also offers students collective psychological support as they learn new content. ELL students gain models for language development; students with disabilities gain models of positive behavior. This support provides all parties, both teachers and students, with a workable socio-cultural compromise between the home culture and culture of the school. By grouping heterogeneously, students with diverse needs can offer and enrich the mainstream students’ learning process with experience and knowledge of their skills.

Make interdisciplinary connections to what students are learning in math.

Using themes in math lessons that can draw interdisciplinary connections will reinforce learning skills from different disciplines. For instance, when showing students the difference between the metric system that they are familiar with and the standard system in the United States, teachers can also teach map skills from social studies class. One way to do this is to create a scenario where students play tourists who have just arrived in the United States. Given a map of the United States, they measure the distance between the airport and their destination using the scales provided, calculate the distance in miles, and convert it to metric. The more opportunities that a student has to connect new knowledge with existing knowledge increases the generalization potential.
Make cultural connections for students when teaching mathematics. Teachers can capitalize on the diverse cultural backgrounds of their students when teaching mathematics. For instance, a world study math center can be set up in the classroom in which students from multicultural backgrounds display the origins of math concepts from their cultures (e.g., algebra [Arabic], geometry [Greek], tangrams [Chinese]). Math and art teachers can find common themes in which they can team teach individual subject matter concepts in their classes.

Rewrite word problems in simple terms. Students with limited language skills, including those who are literate in their first language and students with language-based learning disabilities, often have not mastered the technical vocabulary related to basic math operations. These problems center around concepts and vocabulary such as words of a technical nature such as denominator, quotient, and coefficient and words such as rational, column, and table. By having students work collaboratively in heterogeneous groups to write math problems, teachers perhaps will be able to see how these technical terms might be used or even avoided when students express these problems in their own words. Teachers can also paraphrase and modify some of the more challenging questions by highlighting key terms (Mercer & Mercer, 1998). For instance, in the problem “Five times a number is two more than ten times the number,” students must recognize “a number” and “the number” refer to the same quantity. However, consider this problem: “The sum of two numbers is 77. If the first number is ten times the other, find the number”; students need to know they are dealing with two numbers (Dale & Cuevas, 1992). In the above example, teachers can use pictures and symbols to illustrate the problem, and they can also emphasize the difference that grammatical articles (such as “a” and “the”) can make to the semantics of the problem.

Concretize math concepts with Total Physical Response (TPR; Asher, 1982). TPR is an approach to second language acquisition that is based on the model of how children learn their first language. It can be applied not only to non-native English speakers but also to students with disabilities. In the TPR approach, instructors describe procedures while modeling actions. In math, teachers can use TPR to illustrate problem-solving math questions. For instance, in demonstrating the math concepts of “equal,” “more than,” “less than” (=, >, <), teachers organize two groups of students, with one group containing more students than the other. A more complex TPR activity would be to have the students role play a skit to demonstrate the concept of expansion. Students can line up in a row to play molecules in a rod, while one student plays the fire heating the rod. More students are added to the original row to signify expansion of the rod, and spectators can measure the length of a rod and record the difference. Many other creative activities can be made up using TPR. Students will have a lot of fun learning math using this approach because teachers are involving students in math concepts instead of solely talking about math concepts.

Create word bank charts and hang them in the classroom for viewing. Teaching vocabulary in mathematics instruction is a crucial first step to learning new skills. Creating a literate environment where the classroom is filled with word list charts is important. Teachers can keep word lists for each unit of study and add new words as they appear throughout a unit. Teachers should be encouraged to write mathematics vocabulary both in English and also in the language of their LEP students. In doing this, teachers are encouraging students to maintain and preserve their mother tongue along with seeing and using these new words in English.

Take Internet field trips and use mathematics software. The Internet and computer software are now being used as an instructional tool to explore, investigate, problem solve, interact, reflect, reason, communicate, and learn many concepts that are in U.S. school curricula. Students are able to take virtual tours of places like the Bronx Zoo, the White House, the Louvre Museum and also have access to information from NASA, the United Nations, and so forth. The number of Web sites and educational software available designed for teachers and students.
with the intent to teach concepts is becoming endless. Web sites like funschool.com or www.funbrain.com are ideal for both teachers and students to teach/learn a multitude of math and reading/language concepts K–12, and software like Mathblaster, Tesselmania, and Geometer’s Sketchpad can make the learning of mathematics really dynamic. Many Web sites are designed for teachers, students, and parents and keep track of student achievement and record keeping as well as provide learning in an exciting and interactive way. Today more and more teachers can bring their students to sites to access information or interact and learn concepts on just about any topic under the sun.

Teachers may want to group students in pairs at computers and bookmark the sites for the students in advance. You can also create a worksheet, treasure hunt, or activity sheet with the Web sites listed, along with a guide of questions to answer and activities to do as they visit various sites while taking their Internet Field Trip (Furner, Doan-Holbein, & Scullion-Jackson, 2000).

Many Web sites include the actual pictures of the currently used math manipulatives that are used in classrooms across the country to help reinforce math concepts in a concrete way. Students can explore the geometrical math concept of a tessellation and actually create their own while learning geometry and spatial sense objectives. The possibilities are endless when you infuse the Internet into instruction.

Ameis and Ebenezer (2000) have recently written a book called Mathematics on the Internet in which they provide resources and suggestions for teaching mathematics via the Internet. The book connects math concepts K–12 to many Web sites that can be used to help teach these concepts.

Parents of homeschooled children can benefit from Internet use as a means to learn via Internet Field Trips. The Internet has a definite role to play in the reform of traditional teaching. By using educational software and the Internet, students can now learn in ways that are more exciting and challenging. The Internet also provides students with tools to access information and become independent life-long learners in an age that will increasingly depend on technology to survive in a complex multicultural world.

Use children’s literature to teach mathematics and develop the language. Teachers can reach a child in a non-threatening way by reading children’s literature that helps teach math concepts and connect the learner’s mathematical understanding and at the same time not intimidate, threaten, or turn-off a child to mathematics like some traditional approaches may have. Children’s and adolescents’ literature can be a beneficial way of teaching mathematics.

Benefits of Using Literature in Mathematics
- Teaches math concepts in the context of a story
- Incorporates integrated studies with reading, writing, speaking, listening, and so on
- Develops mathematical thinking
- Prevents math anxiety and creates a less math-anxious classroom environment
- Allows for a variety of responses
- Makes historical, cultural, and practical application connections
- Allows for the use of manipulatives as it relates to the story
- Assesses a child’s understanding by reading/questioning
- Offers a wide range of books that can be used to teach most math concepts K–8
- Leads to problem solving and active involvement from the story’s context
- Provides a shared experience for students and teacher

Teachers can address the NCTM’s “Communication Standard” by incorporating literature in the teaching of mathematics (2000). Teachers can also have students discuss math from the stories and write about such concepts to demonstrate their understanding of math concepts as well as their feelings toward math. In their book Books You Can Count On: Linking Mathematics and Literature, Griffiths and Clyne (1991) wonderfully illustrated many examples of how to connect children’s literature into a math lesson. Here we have included a sample of some of our own suggestions of activities tying literature and writing to mathematics.

Using auditory, visual, and kinesthetic teaching approaches for different learning styles enables teachers to reach more students than the traditional direct-instruction or paper and pencil drill and practice forms of instruction. Some teachers use manipulatives, others allow verbal rehearsals, some may use the new series of math teaching videos by Math Vantage or make math visual using math software or the computer screen. Another option would be to consider kinesthetic learners. These learners learn best by doing. They complete the task by mounting a whole-body effort. Sometimes the overwhelming energy put forth by this type of learner can lead to a loss of focus. Students get off track and miss their objective. One way to meet
the needs of these students is to incorporate refocusing activities as part of the transition process. Moskowitz (1978) suggested that classrooms that offer techniques designed to relax students, increase the enjoyment of learning, raise self-esteem, and blend self-awareness will increase students’ potential. For example, students may need a moment to stop and focus before beginning a group activity. This would involve the whole class or a small group taking a moment to close their eyes and think about what they need to do next. The students do not need words to project what they need to do; they hopefully think through the next step. Learners of varying modalities can focus and plan their next step. Teachers must take into consideration the modalities of their students and try to reach each child regardless of his or her learning style.

Summary

These suggestions are by no means exhaustive. With this grab bag of strategies, teachers can creatively design their lesson activities to meet students’ individual needs. We have offered strategies and techniques that will match the learning styles of all students—reaching all students using teaching styles that adapt to students’ multiple intelligences. With the ESE learners, the use of the multimodal approach that incorporates the multiple intelligence caters to students’ short attention spans as they are not expected to only sit still to learn the materials. With the English Language Learners, we have included the SDAIE (Specially Designed Academic Instruction in English) approach, which supports the teaching of language acquisition while teaching the content area such as math. In all of the 20 strategies, language objectives are incorporated in teaching math concepts; therefore, ELL are given the opportunity to simultaneously develop their English language skills as they are learning math. Math teachers today must work hard to eliminate and prevent any math anxiety their students may develop or carry with them (Furner & Duffy, 2002). Our children today are not only competing for jobs with others in the United States but with others from around the globe who are confident in their ability to do mathematics in this competitively global society. Content area teachers can also play a crucial role in the language learning process of their students, and primary teachers no longer have to be the sole responsible party in teaching ELL the English language; rather, both the content area teachers and the primary teachers can work as a team to help the ELL in their academic work. Each day the diversity of students grows within the confinement of our classrooms, so teachers have to tirelessly keep abreast with their research of diverse teaching strategies to reach all students. Equity in mathematics instruction requires teachers to provide accommodations so everyone in the class can learn mathematics. The best practices mentioned here can assist teachers in reaching all students mathematically.

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