Chapter 13 - Equilibrium and Human Movement

Center of Gravity (CG) or Center of Mass
The point around which the mass and weight of a body are balanced in all direction or the point about which the sum of torques produced by the weights of the body segments is equal to zero.

The Center of Gravity of a symmetrical object of homogeneous density is the exact center of the object.

When mass within an object is not constant, CG shifts in the direction of greater mass.

Center of gravity serves as an index of total body motion.

Locating the Center of Gravity
For one segment objects, CG is the balance point of the three different planes.

As a projectile, the body’s CG follows a parabolic trajectory.

Weight vectors act through the center of gravity (produced the line of gravity).

Segmental Method: procedure for determining total body Center of Mass

\[
X_{COM} = \frac{\sum (x_S) (m_S)}{\sum m_S}
\]

\[
Y_{COM} = \frac{\sum (y_S) (m_S)}{\sum m_S}
\]

The “average” location of the mass of a body

Segment mass and COM locations available in anthropometric tables

Stability and Balance
Equilibrium: An object at rest and will remain at rest.

Stability: The ability of a body to resist a disruption of equilibrium and return to its original state if disturbed (the more stable an object is the more it is able to resist a larger force).

Balance: The ability to control equilibrium.
Factors that Affect Stability

- Mass: Increase in mass will increase stability
- Friction: Increases in friction will increase stability
- Base of Support: Larger bases will increase stability
- Horizontal Position of Center of Gravity: Further from the center (outside the object) will decrease stability
- Height of CG with respect to the base of support: Increase Height of CG will decrease stability

Center of Pressure

Reaction forces between the body and support surface disturbed over the entire contact can be summed into a single point (center of pressure)

Center of pressure: point about which the ground reaction force is balanced

Base of Support: Area enclosed by all points at which the body contacts a supporting surface

Center of gravity can move outside the base of support (acceleration in sprinting) but center of pressure cannot move outside base of support

Static Balance

For equilibrium, the center of pressure must be directly below the body CG, thus the body Center of Mass must be within the boundaries of the base of support

Center of mass must travel a distance before a balance loss; provides more time for deceleration in some cases

Chapter 12-Linear Kinetics of Human Movement

Newton’s Law

Law of Inertia

A body will maintain a state of rest or constant velocity unless acted on by an external force that changes the state

Inertia: is the tendency of an object to maintain its current state of motion, whether motionless or moving at a constant velocity

Inertia in the linear system is directly related to the mass of an object
Law of Acceleration
A force applied to a body causes an acceleration of the body

- Of a magnitude proportional to the force
- In the direction of the force
- And inversely proportional to the body’s mass

\[ F = ma \]

Effects of Force in one dimension
- Net external force & velocity...
  - In same direction: magnitude of velocity increases
  - Opposite directions: magnitude of velocity decreases
- Larger net force \( \Rightarrow \) larger / faster change in velocity

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<thead>
<tr>
<th>Velocity</th>
<th>Force</th>
<th>Change in Velocity</th>
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Effects of Force in two dimension
- A net external force can produce:
  - a change in speed (i.e. in magnitude of velocity)
  - a change in direction
  - a change in both speed and direction

Analyzing Effects of Force in two dimension
- In 2-D, apply Newton’s Laws separately in the x and y directions
- For Newton’s 2\textsuperscript{nd} Law:

\[ \sum \vec{F} = m \vec{a} \quad \iff \quad \begin{cases} \sum F_x = m a_x \\ \sum F_y = m a_y \end{cases} \]
**Law of reaction**
For every action there is an equal and opposite reaction

When one body exerts a force on a second, the second body exerts a reaction force that is equal in magnitude and opposite in direction of the first body

- If body 1 applies a force to body 2, then body 1 experiences a reaction force from body 2:
  - Of the same magnitude
  - At the same point
  - In the opposite direction

**Ground Reaction Force (GRF)**
The magnitude of vertical GRF during level ground running is 2 to 3 times body weight in runners that heel strike

This relates to the striking of the foot during running and how it strikes

- A foot strike infront of the body will cause a GRF that will act against the runners motion and slow down the runner
- A foot strike directly underneath will cause a vertical GRF or even a forward GRF that will help propel the runner forward (especially important during sprinting)

**Friction**
Friction is a force acting over the area of contact between two surfaces in the direction opposite that of motion or motion tendency

Friction is a force so it is a vector and has 1) magnitude 2) direction and 3) point of application
Types of Friction

- Static Friction ($F_s$)
- Maximum Static Friction ($F_s$ max)
- Kinetic Friction ($F_k$)

As long a body is static, the magnitude of the friction force development is equal to that of an applied external force. Once motion is initiated, the magnitude of the friction force remains at a constant level below that of maximum static friction.

\[ F = \mu R \ (or \ N) \]

\( \mu \) can be for static or kinetic friction

\( \mu \) = coefficient or friction: number that serves as an index of interaction between 2 surfaces in contact

\( R \) = normal reaction force: force acting perpendicular to 2 surfaces in contact

Factors influencing \( \mu \)

- Relative roughness and hardness of surfaces
- Type of molecular interaction between surfaces
- Temperature of the surfaces that are interacting (think artificial turf and cleats…increased friction when the temperature warms up which leads to increased injury risk)

Mechanical Behavior of Bodies in Contact

Linear momentum: quantity of motion that is measured as the product of a body’s mass and its velocity

\[ P = mv \]

In the absence of external forces, the total momentum of a given system remains constant

\[ M_1 = M_2 \]

Impulse: product of a force and the time over which the force acts (Impulse=\( Ft \))

\[ Ft = (mv)_2 - (mv)_1 \]
**Impulse and Ground Reaction Force during the Vertical Jump**

Theoretically impulse can be increased by increasing the force applied magnitude or by the increase in time over which the force acts.

This relates to the forces felt in a vertical jump, when someone lands if their knees are bent they will dissipate force of their joints due to an increased time over which the force is acted upon the body.

Fun fact for the day - Arm motion contributes to 12-13% of the total upward momentum for a vertical jump

**Impact**

A collision in which a large force acts over a small time. Force acting during impact has two effects: it is either absorbed and lost through deformation and the remaining force changes the objects directions.

![Impact Diagram](image)

Perfect Elastic: No energy lost. Magnitude of each object’s velocity is the same after impact as before, but direction changes

Perfectly Plastic: Objects deform and stick together

(Most impacts are somewhere in between)

![Elastic and Plastic Diagrams](image)

When two bodies undergo a direct collision, the difference in their velocities immediately after impact is proportional to the difference in their velocities immediately before impact
**Coefficient of Restitution**
The coefficient of restitution measures the elasticity of an impact

COR of 1 is perfectly elastic and 0 is perfectly plastic

COR is determined by the properties of the materials making up both objects

**Hitting a Stationary Object**
If a traveling object strikes a stationary object, depending on the masses, energy will be transferred to the stationary object and will impart a post velocity to the object

- To increase $v_{2\text{post}}$
  - Larger $v_{1\text{pre}}$
  - Larger $m_1$
  - Smaller $m_2$
  - Larger $e$ (i.e. more elastic)

Impact between a moving body and stationary one

$$e = \sqrt{h_b - h_d}$$

$H_b$ = height to which ball bounces

$H_d$ = height from which ball is dropped

(increases in impact velocity and temperature increase the COR)

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**Chapter 14: Angular Kinetics of Human Motion**

**Moment of Inertia**
The Moment of Inertia (I) represents an object's resistance to angular change about some axis

It is the product of each particle’s mass and the radius of rotation for that particle

$$I = mr^2$$

Taller, heavier athletes have greater angular inertia than shorter, light athletes

Moment of Inertia for an Entire Body is the sum of the moments of inertia of all the mass particles

$$I = \sum mr^2$$
**Radius of Gyration**
Distance from the axis of rotation to a point where the body’s mass could be concentrated without altering its rotational characteristics

\[ I = mk^2 \]

\( I = \) moment of inertia/ \( m = \) total body mass/ \( k \) distance know as the radius of gyration (\( k \) is always longer than \( r \))

Radius of Gyration can be a useful index of Moment of Inertia

The axis about which rotation occurs is important to note with radius of gyration.

For example, the radius of gyration (\( k \)) of the forearm for flexion/extension movements is much larger than for pronation/supination

**Principles Moment of Inertia**

Principal moments of inertia of the human body in different positions with respect to different principal axes:

1. Principal axis; 2. Moment of Inertia (kg \( \cdot \) m\(^2\)).

(1) Anteroposterior  
(2) 12.0–15.0

(1) Mediolateral  
(2) 10.5–13.0

(1) Mediolateral  
(2) 4.0–5.0

(1) Longitudinal  
(2) 1.0–1.2

(1) Longitudinal  
(2) 2.0–2.5

**Angular Momentum**
Angular momentum is the quantity of angular motion possessed by a body measured as the product of moment of inertia and angular velocity

\[ H = I \omega \]

\[ H = mk^2 \omega \]

**Conservation of Angular Momentum**
The total angular momentum of a given system remains constant in the absence of external torques

\[ H_1 = H_2 \]

\[ (l \omega)_1 = (l \omega)_2 \]
When angular momentum is conserved, there is a tradeoff between moment of inertia and angular velocity.

- Angular momentum remains constant in the air.
- MOI \((I)\) reduces when the diver goes from a layout to a turck and then increases as the diver opens up to enter the water.
- Angular velocity increases when MOI decreases and it decreases as the diver increases MOI.
- Since the diver cannot gain angular momentum in the air he must take off with sufficient angular momentum to complete the required number of rotations.

**Angular Impulse**

Angular Impulse is the product of torque and the time interval over which the torque acts.

\[
\tau \times t = \Delta H
\]

\[
\tau \times t = (I\omega)_2 - (I\omega)_1
\]

\[
\tau \Delta t = I \Delta \omega
\]
Chapter 13 – Equilibrium and Human Movement

**Moment Arms**

Torque $T = Fd_\perp$ or the rotary effect created by applied force

In the body, the movement arm of muscles is the perpendicular distance between the muscle’s line of pull and the joint center (Largest moment arm at an angle of pull of 90 degrees)

Torque has magnitude and direction (is a vector quantity)

Product of muscle tension and muscle movement arms produce torque at the joint crossed by the muscle

- Much of human movement involves simultaneous tension development in agonist and antagonist muscle groups but the force we examine is net joint torque
- Tension in antagonist controls the velocity of the movement and enhances stability (eccentric training and breaking forces in muscles)
- With other factors constant increased speed with increase resultant joint torques
  - However, increased speed will increase momentum and make technique poorer (technique suffers)

**Levers**

Body movements directly involved in sport and exercise primarily act through the bone levers of the skeleton

Lever - a simple machine with a rigid barlike body that may rotate about an axis

Muscles develop tension which pulls on the bones to support or move a resistance

Bone=lever/ joint=axis of rotation (fulcrum)/ Muscles apply the force
Classification of Levers

First Class Lever: Lever is positioned with the force applied and the resistance on opposite sides of the axis of rotation (fulcrum)

Example: Triceps in the Triceps extension exercise

Second Class Lever: Lever is positioned with the resistance between the force applied & the axis of rotation

Example: Gastrconemius and Standing Heel Raises

Third Class Lever: Lever positioned with the force applied between the axis of rotation and resistance (composes most levers in the human body)

Examples: Biceps & Elbow joint in biceps curl/ Patellar tendon @ knee/ Medial Deltoid @ shoulder

Joints can act as different levers in certain situations such as the elbow joint in the concentric (third class) or eccentric (second class) movements of the biceps curl

Mechanical Advantage
Ratio of the movement arm of the force applied to the moment arm of resistance

A mechanical advantage greater than 1.0 allow the force applied (muscle) to be less than the resistive force to produce an equal amount of torque

A mechanical advantage less than 1.0 is a disadvantage in the common sense of the term

\[
\text{Mechanical Advantage} = \frac{\text{Moment Arm (force)}}{\text{Moment Arm (resistance)}}
\]

First Class Levers has mechanical advantages less than 1.0. So they are disadvantageous

Second Class Levers have mechanical advantages greater than 1.0 (advantageous)

Third Class Levers have mechanical advantages less than 1.0 (disadvantageous)
Most of the skeletal muscles operate at a considerable mechanical disadvantage, thus in sports the force produced by muscles and tendons are much higher than those exerted by the extremities on external objects.

Although at disadvantage mechanically, third class levers in the human body allow for greater range of motion and angular speed (although the force produced by the muscles are extremely high).

**Moment Arm and Mechanical Advantage specifically for the Bicep**

During elbow flexion with the biceps muscle, the perpendicular distance from the joint’s axis of rotation to the tendon’s line of action varies through the range of motion.

As the weight is lifted, the movement arm through which the weight acts, and thus the resistive torque changes with the horizontal distance from the weight to the elbow.

The mechanical advantage is greatest at 90 degrees of elbow flexion.

![Graph showing mechanical advantage vs. elbow angle](image)

**Bullet-brachialis**/ **Square- biceps**/ **Triangle brachioradialis**

**Why Do We Have Knee Caps**

A patella makes the moment arm larger for the knee which allows for a greater ROM and velocity production (compared to knee without a patella). The larger moment arm due to the patella helps increase Mechanical Advantage.

Patellas can also help prevent shear forces due to the femur slipping off the patellar plateau.

**Attachment Site of the Muscle**

When the muscle attaches closer to the axis of rotation, greater velocities will be produced because the bones will travel through greater angles during equal contractions strengths compared to further attachment site from the axis.

When the muscle attaches further from the axis of rotation, greater forces will be produced compared to a muscle that attaches closer to the axis of origin (but will move through less of an angle during a contraction of the same strength).
Weight Lifting Machines and Strength Curves

Machines are made to match the strength curves of muscles.

Moment arms are constantly changing during movement. Machines are built to match strength curves to the peak force production of joint angle.

For example at 90 degrees of elbow flexion is the biceps the strongest, a CAM on a machine will be built to make the machine hardest at 90 degrees to match the strength curve.